

# Thermodynamics, spectral distribution and the nature of dark energy

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Recent astronomical observations suggest that the bulk of energy in the Universe is repulsive and appears like a dark component with negative pressure ( $\omega \equiv p_x/\rho_x < 0$ ). In this work we investigate thermodynamic and statistical properties of such a component. It is found that its energy and temperature grow during the evolution of the Universe since work is done on the system. Under the hypothesis of a null chemical potential, the case of phantom energy ( $\omega < -1$ ) seems to be physically meaningless because its entropy is negative. It is also proved that the wavelengths of the  $\omega$ -quanta decrease in an expanding Universe. This unexpected behavior explains how their energy may be continuously stored in the course of expansion. The spectrum and the associated Wien-type law favors a fermionic nature with  $\omega$  naturally restricted to the interval  $-1 \leq \omega < -1/2$ . Our analysis also implies that the ultimate fate of the Universe may be considerably modified. If a dark energy dominated Universe expands forever, it will become increasingly hot.

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The so-called dark energy or *quintessence* is believed to be the first observational evidence for new physics beyond the domain of the Standard Model of Particle Physics. Its presence, inferred from an impressive convergence of observational results along with some apparently successful theoretical predictions, not only explains the current cosmic acceleration but also provides the remaining piece of information connecting the inflationary flatness prediction with astronomical data [1].

In spite of its fundamental importance for an actual understanding of the evolution of the Universe, the nature of this unknown energy component (which seemingly cannot be unveiled from background tests [2]) constitutes one of the greatest mysteries of modern Cosmology and nothing but the fact that it has a negative pressure (and that its energy density is of the order of the critical density,  $\sim 10^{-29}$  g/cm<sup>3</sup>) is known thus far. This current state of affairs brings naturally to light some important questions (among others) to be answered in the context of this new Cosmology:

1. What is the thermodynamic behavior of the dark energy in an adiabatic expanding Universe? Or, more precisely, what is its temperature law?
2. How do dark energy modes evolve in the course of the cosmic expansion?
3. What is the dark energy frequency spectrum?
4. What is the thermodynamic fate of a dark-energy-dominated universe?

In order to answer the above questions, it is necessary to make some fundamental hypothesis concerning

the unknown nature of the dark energy. In principle, the thermodynamic behavior and the fate of cosmic evolution are somewhat entertained and depend fundamentally on the previous knowledge of the intrinsic nature of the expanding components.

In this *letter* we investigate the thermodynamic and some statistical properties of the dark energy assuming that its constituents are massless quanta (bosonic or fermionic) obeying a constant equation-of-state parameter,  $\omega = p_x/\rho_x$ , where  $p_x$  is the pressure and  $\rho_x$  its energy density. To be more precise, most of the thermodynamic results remains true even if the dark energy medium is formed by massive particles.

In what follows, answers to the above questions are sought by using only the observational piece of information that for dark energy  $\omega$  is a negative quantity. Since  $\omega$  can also be seen as a continuous parameter, the case  $\omega = 1/3$  (blackbody radiation or neutrinos) provides a natural check for our general results. Throughout this paper it will be assumed that the Universe is described by the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) geometry

$$ds^2 = c^2 dt^2 - R^2(t) \left( \frac{dr^2}{1 - \kappa r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right), \quad (1)$$

where  $\kappa = 0, \pm 1$  is the curvature parameter and  $R(t)$  is the cosmological scale factor.

1. *Dark Energy and Thermodynamics.* To begin discussing the first question, we recall that the thermodynamic states of a relativistic simple fluid are characterized by an energy momentum tensor  $T^{\alpha\beta}$ , a particle current  $N^\alpha$  and an entropy current  $S^\alpha$ . By assuming that the dark energy is a perfect fluid such quantities are defined and constrained by the following relations [3]

$$T^{\alpha\beta} = (\rho_x + p_x) u^\alpha u^\beta - p_x g^{\alpha\beta}, \quad T^{\alpha\beta}_{;\beta} = 0, \quad (2)$$

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$$N^\alpha = n u^\alpha, \quad N^\alpha_{;\alpha} = 0, \quad (3)$$

$$S^\alpha = n \sigma u^\alpha, \quad S^\alpha_{;\alpha} = 0, \quad (4)$$

where  $(;)$  means covariant derivative,  $n$  is the particle number density,  $\sigma$  is the specific entropy (per particle) and the quantities  $p_x$ ,  $\rho_x$ ,  $n$  and  $\sigma$  are related to the temperature  $T$  by the Gibbs law

$$n T d\sigma = d\rho_x - \frac{\rho_x + p_x}{n} dn. \quad (5)$$

By taking  $n$  and  $T$  as independent thermodynamic variables and by using the fact that  $d\sigma$  is an exact differential, it is straightforward to show that the temperature evolution law is given by [8] (a dot means comoving time derivative)

$$\frac{\dot{T}}{T} = \left( \frac{\partial p_x}{\partial \rho_x} \right)_n \frac{\dot{n}}{n}, \quad (6)$$

or, equivalently, for  $\omega \neq 0$

$$n = \text{const} T^{\frac{1}{\omega}} \Rightarrow T^{1/\omega} V = \text{const.}, \quad (7)$$

since  $n$  scales with  $V^{-1}$ , where  $V$  is the volume of the considered portion within the fluid. In the case of blackbody radiation ( $\omega = 1/3$ ), the above equations yield the well known results  $n \propto T^3$  and  $T^3 V = \text{const}$ . It means that blackbody radiation cools if it expands adiabatically, a typical behavior for fluids with positive pressure ( $\omega > 0$ ). Physically, this happens because each portion of the fluid is doing thermodynamic work at the expenses of its internal energy. From Eq. (7), however, we find that dark energy becomes hotter in the course of the cosmological adiabatic expansion since its equation-of-state parameter is a negative quantity. A physical explanation for this behavior is that thermodynamic work is being done on the system (see Fig. 1). In particular, for the vacuum state ( $\omega = -1$ ) we obtain  $T \propto V$ . Therefore, any kind of dark energy with negative pressure (including the so-called phantom energy,  $\omega < -1$  [4, 5, 6]) becomes hotter if it undergoes an adiabatic expansion. Naturally, this unexpected result needs to be explained from a more fundamental (microscopic) viewpoint, which is closely related to the remaining questions. However, before discussing these questions, it is worth noticing that by combining Eqs. (2) and (7) one obtains the generalized Stefan–Boltzmann law (see [7] for some independent derivations)

$$\rho_x(T) = \eta_\omega T^{\frac{1+\omega}{\omega}}, \quad (8)$$

where  $\eta_\omega$  is a  $\omega$ -dependent constant. For  $\omega = -1$  one finds  $\rho_x = \text{constant}$ , as expected for the cosmological

## Dark Energy: Thermodynamics

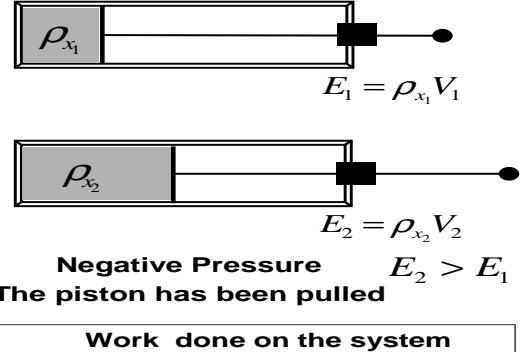


FIG. 1: Dark Energy and thermodynamic work. The total energy and temperature of the dark component grow in an adiabatic expansion because work has been done on the system ( $E_x \equiv \rho_x V \propto T$ ). From Eqs. (7), (8) and (9) we see that a vacuum filled Universe ( $\omega = -1$ ) is a limiting case where the temperature increases, the energy density remains constant, and the entropy is zero.

constant case. The entropy of this  $\omega$ -fluid is also of interest. If the chemical potential is null (as occurs for  $\omega = 1/3$ ), the expression  $\sigma = S_x/N = (\rho_x + p_x)/nT$  defines its specific entropy [9]. Therefore, the dark energy entropy can be expressed as

$$S_x(T, V) = \eta_\omega (1 + \omega) T^{1/\omega} V, \quad (9)$$

from which the adiabatic temperature evolution law (7) is readily recovered. The above result completes our thermodynamic description for the dark component. Note that the vacuum entropy is zero ( $\omega = -1$ ) whereas for phantom or *supernegative* dark energy ( $\omega < -1$ ) the entropy assumes negative values being, therefore, meaningless. If the hypothesis of a null chemical potential is reasonable, this latter result poses a serious problem to a phantom energy description with basis on the usual XCDM parameterization and introduces a new thermodynamic lower limit to the dark energy equation-of-state parameter, i.e.,  $\omega \geq -1$ . Such a limit is in fully agreement with theoretical arguments that motivate the so-called  $\Lambda$  barrier[5] and goes against several observational analyses which seem to favor phantom cosmologies over  $\Lambda$ CDM or usual quintessence scenarios[4, 5, 6]. It should be recalled that thermodynamic states with negative pressure are metastable and usually connected with phase transitions (the most known example at the level lab is an overheated Van der Waals liquid [10]). It should be stressed, however, that the thermodynamic behavior derived here holds regardless of the specific physical mechanism responsible for the negative pressure.

## 2. Effects of the Expansion on the Dark Energy Modes.

In what follows, we hypothesize that the constituents of the dark energy are massless particles (bosons or fermions) and that the differences between this *fluid* and blackbody radiation (or neutrinos) are macroscopically quantified by the equation-of-state parameter  $\omega$ . As we shall see, this hypothesis leads to a microscopic picture in harmony with the above thermodynamic behavior.

The adiabatic theorem [11] guarantees that if a hollow cavity (e.g., our Universe) containing dark energy changes adiabatically its volume, the ratio between the energy of a given mode and the corresponding frequency remains constant, that is,  $E_\nu/\nu = \text{const.}$ , for any proper oscillation. Thus, if  $\rho_x(T, \nu)$  is the spectral energy density inside an enclosure with volume  $V$  at temperature  $T$ , this adiabatic invariant guarantees that  $\rho_x(T, \nu)d\nu V/\nu = \text{const.}$  Now, recalling that the energy density in the band  $d\nu$  varies with the temperature in the same manner as the total energy density, one may write [see (8)]

$$T^{\frac{1+\omega}{\omega}}V/\nu = \text{const.} \quad (10)$$

and, since  $T^{\frac{1}{\omega}}V = \text{const.}$ , it follows that  $T/\nu$  is an invariant. Therefore, whether a hollow cavity containing dark energy is expanding adiabatically, the wavelength of each mode satisfies

$$\lambda T = \text{const.} \quad (11)$$

The above results allow us to understand from a microscopic viewpoint why the net energy of a dark component increases during the expansion. As shown in Figure 2, the wavelengths of any radiative fluid with positive pressure ( $\omega > 0$ ) increase in virtue of the expansion thereby lowering the total energy in accordance with the generalized Stefan–Boltzmann law. However, if  $\omega < 0$ , we have seen that the temperature grows whether the fluid undergoes an adiabatic expansion [see Eq.(7)], and this is accompanied by a decreasing in each wavelength  $\lambda$  in order to satisfy the above relation. This, therefore, constitutes a microscopic explanation of why the energy in a given portion of the dark component increases (see Figure 1). In particular, the vacuum state, i.e.,  $\omega = -1$ , behaves as a limiting case for which the total energy increases during the expansion while its energy density remains constant.

*3. The spectrum of dark energy.* Let us now quantify the influence of a negative pressure on the general form of the spectral distribution. This is an important point because relations (7) and (8) must be recovered from the frequency spectrum. To that end, we consider again an enclosure containing dark energy at temperature  $T_1$  and focus our attention on the band  $\Delta\lambda_1$  centered on the wavelength  $\lambda_1$  whose energy density is  $\rho_x(T_1, \lambda_1)\Delta\lambda_1$ . If the temperature  $T_1$  changes to  $T_2$  due to an adiabatic expansion, the energy of the band changes to  $\rho_x(T_2, \lambda_2)\Delta\lambda_2$  and, according to Eq.(11),  $\Delta\lambda_1$  and  $\Delta\lambda_2$  are related by  $\Delta\lambda_2/\Delta\lambda_1 = T_1/T_2$ . Now, since one can assume that dis-

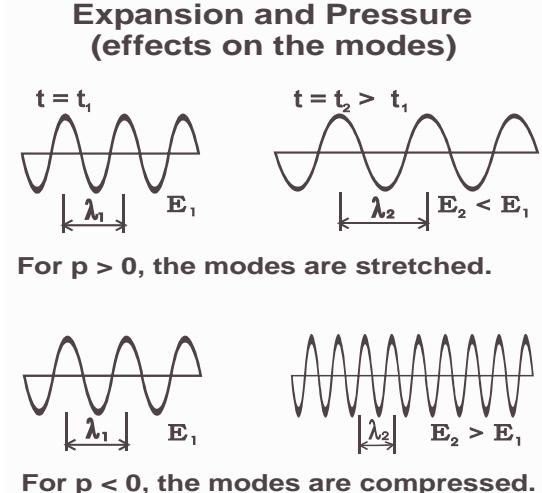


FIG. 2: Pressure effect on the wavelength at two different times ( $t_1, t_2$ ) in an expanding Universe. The wavelengths from radiation with positive (negative) pressure increase (decrease) in the course of the expansion. Energy is continuously stored in the dark energy modes ( $p < 0$ ). This unexpected behavior is a consequence of Eqs. (7) and (11). It explains why the internal energy of the dark radiation increases during the evolution of the Universe.

tinct bands do not interact, it follows that

$$\frac{\rho_x(T_2, \lambda_2)\Delta\lambda_2}{\rho_x(T_1, \lambda_1)\Delta\lambda_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\omega+1}{\omega}}. \quad (12)$$

By combining the above results, and using again the adiabatic invariant (11), we obtain for an arbitrary component  $\rho_x(T, \lambda) = \alpha \lambda^{-\frac{1+2\omega}{\omega}} \phi(\lambda T)$ , or still, in terms of frequency

$$\rho_x(T, \nu) = \alpha \nu^{\frac{1}{\omega}} \phi\left(\frac{\nu}{T}\right), \quad (13)$$

where  $\alpha$  is a constant with dimension of [Energy.Lenght $^{-3}$ .Time $^{(1+\omega)/\omega}$ ], and  $\phi$  is an arbitrary dimensionless function of its argument. The above expression is the generalized Wien-type spectrum for dark energy.

The specific form of the dimensionless function  $\phi(\nu/T)$  can be determined using different approaches, among them, the arguments put forward by Einstein in his original deduction of the blackbody distribution [12]. For a massless bosonic gas, for instance,  $\phi(\nu/T)$  is exactly the occupation number (without chemical potential), and the spectrum takes the final form  $\rho_x(T, \nu) = \alpha \nu^{\frac{1}{\omega}} \times [e^{\hbar\nu/k_B T} - 1]^{-1}$  (see [13] for a more detailed discussion). Therefore, in order to include the case of fermions, we write the spectral distribution (13) as

$$\rho_x(T, \nu) = \frac{\alpha \nu^{\frac{1}{\omega}}}{e^{\hbar\nu/k_B T} \pm 1}, \quad (14)$$

which is the most natural extension of a Planck (or Fermi-Dirac) spectrum describing the  $\omega$ -family of dark energy *radiation*. Einstein's derivation follows for  $\omega = 1/3$  [14] and, more important, the macroscopic relations (7) and (8) are recovered from the above spectral distribution, i.e.,

$$n(T) = \int_0^\infty \frac{\rho_x(T, \nu)}{h\nu} d\nu = \bar{\eta}_\omega T^{\frac{1}{\omega}}, \quad (15)$$

and

$$\rho_x(T) = \int_0^\infty \rho_x(T, \nu) d\nu = \eta_\omega T^{\frac{1+\omega}{\omega}}, \quad (16)$$

where the pair of constants  $(\bar{\eta}_\omega, \eta_\omega)$  depend on  $\alpha$ ,  $k_B$  and  $\omega$ , and also on the underlying statistics obeyed by the dark quanta.

Another interesting point is related to the wavelength for which the distribution (14) attains its maximum value. It is determined by the algebraic condition

$$e^{-x} \pm 1 = \pm \omega(1 + 2\omega)^{-1}x, \quad (17)$$

where  $x = hc/k_B\lambda T$  and the signs  $\pm$  stand for the fermionic and bosonic cases, respectively. The displacement Wien's law now reads

$$\lambda_m T = \frac{hc}{k_B x'(\omega)} = \frac{1.438}{x'(\omega)}, \quad (18)$$

where  $x'(\omega)$  is the root of the above transcendent equation. As one may check, in the case of bosons, a large set of nontrivial solutions exist with positive  $\omega$ , but all of them are out of the dark branch ( $\omega < 0$ ). However, by assuming that the basic constituents of dark energy are massless fermions, the existence of solutions fixes the upper limit  $\omega < -0.5$ . In particular, for  $\omega = -2/3$ , a value compatible with many astrophysical data [1, 16], we find  $\lambda_m T = 1.943$  cm.K while for  $\omega = -1$ ,  $\lambda_m T = 1.123$  cm.K.

*4. Dark energy and the thermodynamic fate of the expanding Universe.* The fate of our Universe, i.e., whether it will eventually re-collapse and end with a Big Crunch, or will expand forever, is a matter of great interest among cosmologists (see, e.g., [15]). In the standard view, if the Universe expands forever it will become increasingly empty and cold. However, whether our analysis provides a realistic description of the the dominant dark energy component, such a destiny is not so neat because of its strange thermodynamic behavior.

In the context of a FRW-type geometry ( $V \propto R^3$ ), Eq. (7) implies that  $T \propto R^{-3\omega}$  or, equivalently, that the general redshift-temperature law reads  $T_x(z) = T_x^o(1+z)^{3\omega}$ , where  $T_x^o$  is the present value of the dark energy temperature. As discussed earlier,  $T_x(z)$  grows in the course

of the expansion while the observed average temperature of the Universe is given by the decreasing temperature of the CMB photons. Therefore, an important point for the thermodynamic fate of the Universe is to know how long the dark energy temperature will take to become the dominant temperature of the Universe. A basic difficulty, however, is that the present-day dark energy temperature has not been measured and, moreover, the  $\alpha$  parameter appearing in the spectrum [Eq. (14)] (or, equivalently, the  $\eta_\omega$  in the expression of the energy density) is still unknown [17]. A very naive estimate of  $T_x^o$  can be done by considering the observational evidence for the density parameters,  $\Omega_x \sim 0.7$  and  $\Omega_r \sim 10^{-4}$  (radiation). For  $\omega \neq -1$  one finds  $T_x^o \sim (10^5 a/\eta_\omega)^{\frac{\omega}{1+\omega}}$ , where  $a$  is the radiation constant. Therefore, any estimate of the present dark energy temperature depends crucially on the ratio parameter  $a/\eta_\omega$ . In particular, for  $\omega = -2/3$  we find  $T_x^o \sim 10^{-10}(\eta_{-2/3}/a)^2$ , so that temperatures of the order of  $10^{-6}$ K are easily obtained for  $\eta_{-2/3}/a \sim 10^2$ . Albeit many aeons might be necessary for the Universe to enter in the dark energy temperature regime, the only possible conclusion is that the ultimate fate of the Universe may be considerably modified: a dark energy dominated Universe expanding forever will become increasingly hot.

Summarizing, we have proposed that the constituents of dark energy are massless particles (bosons or fermions) whose collective behavior resembles a kind of radiation fluid with negative pressure. Through a thermodynamic analysis and basic statistical considerations we derived some relevant properties obeyed by this mysterious component. As expected for a consistent treatment, all thermodynamic relations are recovered from the statistical approach. Perhaps, more important, with basis on a Wien-type law it was possible to show that a fermionic nature is clearly favored because there are nontrivial solutions within the dark energy branch ( $\omega < 0$ ). It was also proved that the existence of such solutions requires  $\omega$  to be  $< -0.5$ . When combined with the limit obtained from the entropy calculation [Eq. (9)], this result constrains the equation-of-state parameter to the interval  $-1 \leq \omega < -0.5$ , which is surprisingly close to the range required from many different astrophysical data [1, 16]. Such results means that some kind of massless particles obeying the Fermi-Dirac statistics should be considered among the candidates for dark energy proposed in the literature. However, it should be stressed that our treatment does not eliminate the case of massive fermions for which most of the thermodynamic properties discussed here remains true.

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